Calculus Warm Up #2-5

No Grapher. Do by hand.

Find the derivative:

1. $y = \dfrac{(x - 5)^3}{x^2}$

2. $y = \dfrac{\pi}{(3x)^2}$

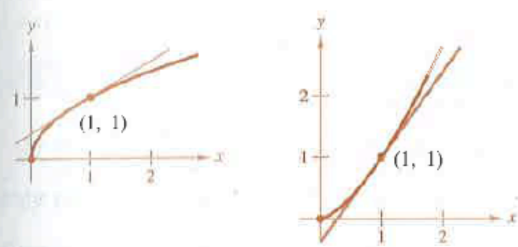
3. Find the point-slope equation of the line tangent to

$y = - x^4 + 5x^2 - 2$ at $x = - 1$

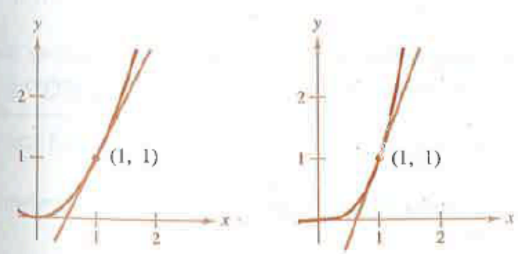
HW Questions:

In Exercises 1 and 2, find the slope of the tangent line to $y = x^n$ at the point $(1, 1)$.

1. (a) $y = x^{1/2}$ (b) $y = x^{3/2}$



(c) $y = x^2$ (d) $y = x^3$



In Exercises 3–12, find the derivative of the given function.

3. $y = 3$

5. $f(x) = x + 1$

7. $g(x) = x^2 + 4$

9. $f(t) = -2t^2 + 3t - 6$

11. $s(t) = t^3 - 2t + 4$

In Exercises 13–18, find the value of the derivative of the given function at the indicated point.

	Function	*Point*
13.	$f(x) = \frac{1}{x}$	$(1, 1)$
15.	$f(x) = -\frac{1}{2} + \frac{7}{5}x^3$	$\left(0, -\frac{1}{2}\right)$
17.	$y = (2x + 1)^2$	$(0, 1)$

In Exercises 19–30, find $f'(x)$.

19. $f(x) = x^2 - \frac{4}{x}$

21. $f(x) = x^3 - 3x - \frac{2}{x^4}$

23. $f(x) = \frac{x^3 - 3x^2 + 4}{x^2}$

25. $f(x) = x(x^2 + 1)$

27. $f(x) = x^{4/5}$

29. $f(x) = \sqrt[3]{x} + \sqrt[5]{x}$

In Exercises 31–36, complete the table, using Example 6 as a model.

Function	Rewrite	Derivative	Simplify
31. $y = \frac{1}{3x^3}$			
33. $y = \frac{1}{(3x)^3}$			
35. $y = \frac{\sqrt{x}}{x}$			

$= x^{1/2} \cdot x^{-1}$

$x^{\frac{1}{2} - \frac{2}{2}}$

$y = x^{-1/2}$

$y' = -\frac{1}{2}x^{(-1/2)-1}$

$-\frac{1}{2}x^{-3/2}$

$-\frac{1}{2x^{3/2}}$

In Exercises 37 and 38, find an equation of the tangent line to the given function at the indicated point.

37. $y = x^4 - 3x^2 + 2$, $(1, 0)$

In Exercises 39–42, determine the point(s) (if any) at which the given function has a horizontal tangent line.

$y' = m = 0$

39. $y = x^4 - 3x^2 + 2$

41. $y = \dfrac{1}{x^2}$

$y = x^{-2}$ $\qquad y' = -2x^{-3}$

$0 \neq -\dfrac{2}{x^3}$

3.2 Rates of Change

-Straight line motion

-Average velocity

-Instantaneous velocity

-Acceleration

-Higher order derivatives

Straight-line motion:
"A common use of a rate of change is to describe the motion of an object moving in a straight line. Such motion is called rectilinear motion. It is customary to use either a horizontal or vertical line with a designated origin to represent the line of motion. Movement to the right (or upward) is considered to be in the positive direction, and movement to the left (or downward) is considered to be in a negative direction.

The function *s* that gives the position (relative to the origin) of an object as a function of time *t* is called a position function." Larson

s(t)
position of an object over time

Definition of Average Velocity:

Over an interval

If $s(t)$ gives the position at time t of an object moving in a straight line, then the average velocity of the object over the interval $[t, t + \Delta t]$ is given by

$s(t)$ $s(t+\Delta t)$
t Δt $(t+\Delta t)$

$$\text{Average velocity} = \frac{\Delta s}{\Delta t} = \frac{\overbrace{s(t+\Delta t) - s(t)}^{\text{change in position}}}{\underbrace{\Delta t}_{\text{change in time}}}$$

Remember:

d=r·t, so $r = \frac{d}{t}$

Find the average velocity of the given position function over the given intervals → time intervals

$s = -16t^2 + 100$

1. [1, 2] 2. [1, 1.5] 3. [1, 1.1]

$$\text{Avg Vel} = \frac{s(2) - s(1)}{2 - 1} = \frac{-64 + 100 - (-16 + 100)}{1} = 36 - 84 = -48$$

$$\text{Avg Vel} = \frac{S(1.5) - S(1)}{1.5 - 1}$$

Definition of (Instantaneous) Velocity:
If $s = s(t)$ is the position function for an object moving along a straight line, then the velocity of the object at time t is given by

The rate of change in position over time.

at a particular point

$$v(t) = \boxed{\lim_{\Delta t \to 0} \frac{s(t+\Delta t) - s(t)}{\Delta t}} = s'(t)$$

Find the velocity of the given position function at the given times

$s = -16t^2 + 100$

$v(t) = s'(t) = -32t$

1. $t = 1$ 2. $t = 2$

$v(1) = -32(1)$
$= -32$

$v(2)$
$= -32(2)$
$= -64$

Note the difference:

Velocity has direction. (Positive or Negative)

Speed is the absolute value of velocity.

Definition of Acceleration:
If s is the position function for an object moving along a straight line, then the acceleration of the object at time t is given by

$a(t) = v'(t)$

where $v(t)$ is the velocity at time t

Find the acceleration of a free-falling object whose position function is

$s = -16t^2 + 100$

$v(t) = s'(t)$
$v(t) = -32t$
$a(t) = v'(t) = s''(t)$
$= -32$

Acceleration:

The rate of change in velocity over time.

Higher-order derivatives

To derive the acceleration function we must differentiate the position function twice

$$\boxed{\begin{aligned} &s(t) \\ &s'(t) = v(t) \\ &s''(t) = v'(t) = a(t) \end{aligned}}$$

We call $a(t)$ the second derivative of $s(t)$. The second derivative is an example of a higher-order derivative.

First derivative:	y',	$f'(x)$,	$\dfrac{dy}{dx}$,	$\dfrac{d}{dx}[f(x)]$,	$D_x(y)$
Second derivative:	y'',	$f''(x)$,	$\dfrac{d^2y}{dx^2}$,	$\dfrac{d^2}{dx^2}[f(x)]$,	$D_x^2(y)$
Third derivative:	y''',	$f'''(x)$,	$\dfrac{d^3y}{dx^3}$,	$\dfrac{d^3}{dx^3}[f(x)]$,	$D_x^3(y)$
Fourth derivative:	$y^{(4)}$,	$f^{(4)}(x)$,	$\dfrac{d^4y}{dx^4}$,	$\dfrac{d^4}{dx^4}[f(x)]$,	$D_x^4(y)$
$\vdots$					
nth derivative:	$y^{(n)}$,	$f^{(n)}(x)$,	$\dfrac{d^ny}{dx^n}$,	$\dfrac{d^n}{dx^n}[f(x)]$,	$D_x^n(y)$

HW: p. 111

1 - 29 odd

Quiz next Wed

Limit definition of the derivative

Find derivatives with the power rule

Find the equation of a tangent line